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MUON BACKGROUNDS PRODUCED BY MULTIPLE BEAM CROSSINGS IN POPAE STRAIGHT SECTIONS

J. Sandweiss
Yale University

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Introduction

Multiple beam crossings in colliding-beam straight section designs are potentially desirable if a straight section area is used for more than one experimental interaction region. This would permit substantial economy in the sharing of utilities and services, of access roads, and of access devices (elevators, ramps, etc.) to the experimental areas(1).

Also, in the insertion scheme proposed by W.W. Lee and L.C. Teng(2), the beams may be sufficiently close at certain points before and after the experimental crossing, that additional collision regions of low luminosity can occur in the straight section. These points will be referred to as "auxiliary crossings" below, and it will be assumed that their luminosity may be as high as 10% of that for the experimental crossing.(3)

The principal disadvantage of multiple crossings is the radiation background which one (or the other) crossing produces at the site of the experimental crossings. The purpose of this note is to discuss these backgrounds generally and to present detailed estimates for one component, the muons.

The backgrounds produced by a crossing can be enumerated as follows:

1. Pions directly produced in the crossing
2. Muons from the decay of the primary pions,

3. Hadrons produced in the vacuum wall and shields by the primary pions

4. Elastically scattered and quasi-elastically scattered protons from the crossing,

5. Hadrons produced by the protons of item 4 striking the vacuum wall area shields.

Briefly, the following comments concerning these different sources of background can be made:

1. For any reasonable design, there will be sufficient room to shield the neighboring experiments from the direct pions and to essentially completely absorb the hadronic shower initiated when the pions strike the shield. As an example, the nuclear absorbtion length in iron is 17.1 cm and typical shield thicknesses will vary from 25 to 50 meters, providing 146 to 292 interaction lengths.

2. The direct pions will have a substantial flight path in the POPAE vacuum chamber, typically 15 to 30 meters, before striking the vacuum wall and shield. The resulting muons are extremely penetrating and present a substantial background possibility. The bulk of this paper discusses detailed estimates of this muon background.

3. The hadronic shower initiated by the primary pion, as has been discussed above, appears to present no serious background problem.

4. The principal difference between the elastic (and quasi-elastic) protons and the direct pions is their respective energy spectra. The pions are produced at relatively low energies, the bulk of them with less than half the incident beam energy. Because of their low energy the focusing magnets of the insertion rapidly

defocus the beam into the wall, a feature which allows most of the insertion length to be used for shielding purposes. The protons on the other hand are scattered at or near the beam energy and will travel most if not all of the distance down the straight section. They, therefore, may strike the wall in close proximity to the "next" experiment and are a potential source of background. The scattered protons may, however, remain inside the vacuum chamber for the whole length and thus present little background problem. This subject requires careful analysis and will not be discussed further in this report.

5. The comments in item 4 above apply as well to the hadronic shower initiated by the scattered protons striking the wall and shield.

Calculation of the Background Muon Rate

The calculations presented here are for a 1000 GeV x 1000 GeV colliding beam situation. Clearly particle production cross sections are unknown for this situation and can at best be estimated. We use the pion production formula proposed by C. Wang[4] which for x, ($x = p_\pi/p_{max}$), values greater than ~ .05 fits all known data, including ISR data to within a factor of 2. As will be noted we shall use the formula for $x \gtrsim .05$ and therefore expect it to be a reasonable approximation. Accordingly,

$$\frac{d^2\sigma}{d\Omega dp} = AP_M\, x(1-x)\, \exp\{-Bx^c - DP_T\} \qquad (1)$$

where the cross section has units of Mb/Steradian/GeV/c, and

P_M = The maximum momentum (in GeV/c),

$x = P/P_M$, P = momentum of produced pion,

P_T = Transverse momentum = $P\theta$.

The constants are given in Table I:

Table I

Values of empirical constants in Wang formula

	A	B	C	D
π^-	51.403	5.732	1.333	4.247
π^+	77.793	3.558	1.333	4.727

Integrating equation (1) over solid angle yields:

$$\frac{d\sigma}{dp} = \frac{2\pi A}{D^2 \; P_M} \left(\frac{1-x}{x}\right) \exp\{-BX^c\} \tag{2}$$

Formula (2) has been evaluated for the π^+ and π^- spectra and the results are presented in figures 1 and 2. It is often of interest to know the integrated cross section for producing a pion of any momentum greater than P. This has also been calculated via numerical integration of (2) and is presented in figure 3.

As a typical illustration of a POPAE straight section design using multiple crossings we take the low β insertion of reference (2) and replicate it once. That is, take two low β insertions in series as illustrated in figure 4.

The four auxiliary crossings near the ends of each section and the two experimental crossings are marked on the figure. Estimates(3) of the drift distance of low energy particles from the experimental crossing indicate a drift space of ~ 30 meters. At the auxiliary crossings each beam has an angle of ~ 5 mr relative to the longitudinal axis of the straight section and the pions reach the wall in a correspondingly shorter distance. A space of 14.5 meters has been taken below for the average drift space for pions from the auxiliary crossings.

If the two 28.5 and two 10.6 meter spaces (between quadrupoles) between the two experimental crossings were all available for shielding a shield length of 78 meters would be available. It seems reasonable that in most cases at least 50 meters would be available for shielding. In some cases the experiments may require a reconfiguration of the insertion but based on studies of insertion design for other storage ring projects it seems reasonable to assume that 25 meters will be a minimum shield thickness. The above geometry is summarized in Table II.

Table II

Straight Section Geometry Assumptions

	Length (meters)
Shield thickness between experimental crossings, typical	50
Shield thickness between experimental crossings, worst case	25
Shield thickness between experimental crossings, best case	78
Free drift space for pions from experimental crossing	30

	Length (meters)
Free drift space for pions from auxiliary crossing	15
Shield thickness between auxiliary crossing and experimental crossing	20

These values are clearly dependent on the details of the straight section and experiment design. However, the values in Table II should be roughly right and some reasonable assumptions are necessary to estimate the background. The sensitivity of the results to these assumptions will be illustrated by their variation for the different conditions listed in Table II.

For relativistic energies, a pion of momentum P_π produces a decay muon spectrum which extends from .57 P_π to P_π for the muon momentum and is uniform over that range.

$$\frac{dN_\mu}{dP_\mu} = \frac{L_D \; M_\pi c}{.43 \; P_\pi^{\;2} \; (c\tau)_\pi} \quad \text{for } .57 \; P_\pi < P_\mu < P_\pi$$

where $\frac{dN_\mu}{dP_\mu}$ $\equiv$ Muons per unit momentum from a single pion of momentum P_π traveling a decay distance L_D meters

$(c\tau)_\pi$ = 7.8 meters

$M_\pi c$ = .14 GeV/c .

If the pions have a momentum spectrum $\frac{dN_\pi}{dP_\pi}$ (= no. of pions per unit pion momentum) then the muon spectrum is given via:

$$\frac{dN_\mu}{dP_\mu} = .0417 \; L_D \int_{P_\mu}^{1.754 \; P_\mu} \frac{1}{P_\pi^{\;2}} \; \frac{dN_\pi}{dP_\pi} \; dP_\pi \tag{3}$$

Formula (3) assumes all pions have the same decay path L_D. A more exact calculation would include the variation of L_D with pion momentum.

It is convenient to define a differential and integral cross section for producing muons which is suggested by equation (3), namely:

$$\frac{d\sigma_\mu}{dP_\mu} = .0417\ L_D \int_{P_\mu}^{1.754\ P_\mu} \frac{1}{P_\pi{}^2} \left(\frac{d\sigma_{\pi^+}}{dP_\pi} + \frac{d\sigma_{\pi^-}}{dP_\pi} \right) dP_\pi \qquad (4)$$

Since we do not care for background calculations whether the muon is positive or negative we have used the sum of π^+ and π^- cross sections in equation (4).

Finally, we define an integral cross section for producing a muon of any momentum greater than P via

$$\sigma_\mu\ (P_\mu > P) = \int_P^{1000} \frac{d\sigma_\mu}{dP'}\ dP' \qquad (5)$$

The quantity $\frac{1}{L_D}\ \sigma_\mu\ (P_\mu > P)$ is independent of the drift space for pion decay and depends only on the pion production spectrum and decay kinematics. Equation (4) has been numerically calculated using the $\pi^\pm$ spectra given in figures 1 and 2 and the resulting $\frac{1}{L_D} \frac{d\sigma_\mu}{dP_\mu}$ has been numerically integrated to calculate $\frac{1}{L_D}\ \sigma_\mu\ (P_\mu > P)$. The result is presented in figure 5.

We have calculated the background muon rate for four cases corresponding to various combinations of drift space shield thickness and luminosity (see Table II). The results are presented in Table III.

Table III

Muon Background Rater

Case	Luminosity (cm^2 sec^{-1})	Shield Thickness of Iron (Meters)	Drift Space (Meters)	Muon Rate at "next" Experimental crossing (Muons/Sec)
I	10^{34}	50	30	6.75×10^5
II	10^{34}	25	30	2.28×10^6
III	10^{34}	78	30	3.36×10^5
IV	10^{33}	20	15	1.74×10^5

As an illustration of the method, we explicitly calculate Case III. The muon energy needed to penetrate a 78 meter shield is [5] 127 GeV. From figure 5:

$$\sigma_\mu \; (P_\mu > 127 \text{ GeV/c}) = L_D \times 112 \times 10^{-5} \text{ mb}$$

$$= 30 \times 112 \times 10^{-5} = .0336 \text{ mb}$$

$$\text{Muon Rate } (P_\mu > 127 \text{ GeV/c}) = 10^{34} \times .0336 \times 10^{-27}$$

$$= 3.36 \times 10^5 \text{ muons per sec.}$$

Cases I, II, and III correspond to backgrounds produced by one experimental crossing at the next experimental crossing for the assumptions listed in Table II. Case IV corresponds to backgrounds produced by the Lee, Teng[2] auxiliary crossing (at the next experimental crossing).

The diameter of the shield must be sufficient to contain the muons, considering both their production angles and their multiple coulant scattering in the shield. We make an approximate estimate as follows.

From formula (1) it is easy to show that the fraction, r, of pions produced with transverse momentum $P_T > P_T^*$ is given by:

$$r = e^{-D\, P_T^*}(1+D\, P_T^*) \qquad (6)$$

From equation (6) we easily see that 95% of the pions (and hence the muons) are produced at a $P_T^* < 1.05$ GeV/c. We furthermore note that for a 50 meter shield the rms transverse momentum due to multiple coulomb scattering is 0.8 GeV/c. We thus estimate that 95% of the muons which are stopped in the shield have $P_T < 1.85$ GeV/c. To convert this into an angle and thus to a shield diameter we take a 100 GeV/c pion or the "average" muon parent. The distance of the end of the 50 m shield for the straight-section design illustrated in figure 4 is 126 meters so we may estimate the shield diameter as follows:

$$\text{Diameter of 50 m Shield (5\% leakage)} = 2 \times 126 \times \frac{1.85}{100} = 4.7 \text{ meters} .$$

Clearly, the above calculation should be regarded only as setting the rough scale for the shield size.

In a similar way we may estimate the size of the muon flux distribution at the "next" experiment.

$$\text{Diameter of Muon Flux field containing} \sim 95\% \text{ of the Muons} = 2 \times 150 \times \frac{1.85}{100} = 5.6 \text{ meters}$$

Conclusion

The background rates given in Table III are large enough so that for many experiments they would be a significant factor. For example, a spark chamber or streamer chamber system operating close to the interaction with the purpose of detecting a majority of the particles from the interaction would "see" a background muon in Case I on almost every picture. This may or may not be serious, depending on the particular experiment. It is very likely that in each individual experiment, methods can be devised to "live with" such a muon background. The question is, rather, whether the combined cost of all the background "fixes" wipes out the original gain in economy due to using multiple crossings. It is also clear that if the POPAE luminosity is 10^{33} (maximum) rather than 10^{34} the muon background from multiple crossings move into the tolerable range for most experiments.

Finally, it should be emphasized that these calculations are approximate and involve many assumptions. What they do indicate is that such muon backgrounds are a potentially important factor which should be studied with much greater care before a multiple crossing design is adopted.

References

1. D. Edwards and T. Collins, Private communication
2. W.W. Lee and L.C. Teng, "Insertions for Colliding Beam Storage Rings", *Proceedings of the IX-th International Conference on High-Energy Accelerators*
3. L.C. Teng, Private communication
4. C.L. Wang, "On Pion Production in High-Energy Collisions", Brookhaven National Laboratory Report 18798
5. D. Theriot, "Muon $\frac{dE}{\phi x}$ and Range Tables: Results for Shielding Materials Using Collision Losses Only", National Accelerator Laboratory Report TM-260, 1111.11, July 21, 1970

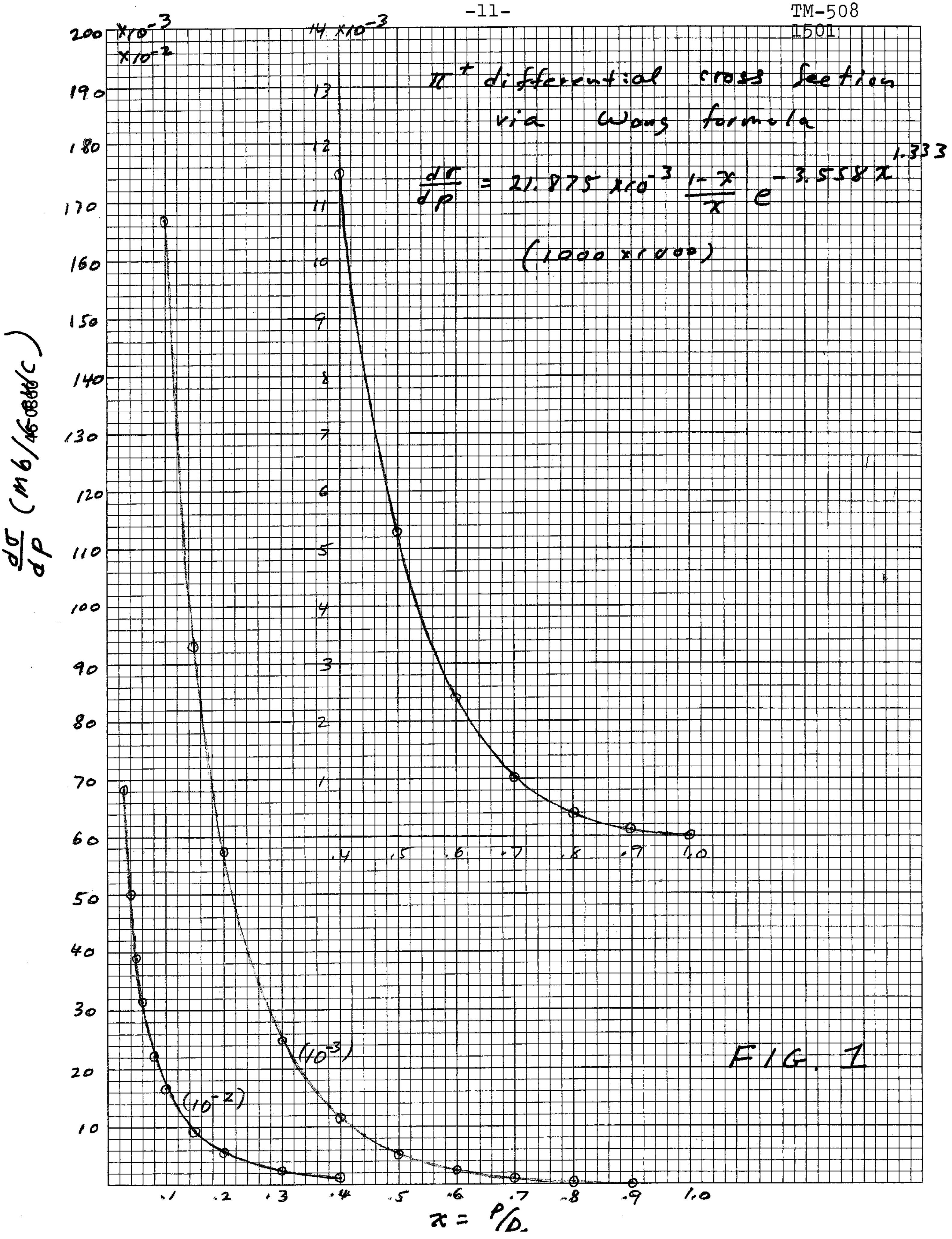

π^+ differential cross section
via Wong formula
$\frac{d\sigma}{dp} = 21.875 \times 10^{-3} \frac{1-x}{x} e^{-3.558 x^{1.333}}$
(1000 x 1000)
$\frac{d\sigma}{dp}$ (mb/GeV/c)
$x = P/P_0$
$\times 10^{-3}$
$\times 10^{-2}$
14×10^{-3}
(10^{-3})
(10^{-2})

FIG. 1

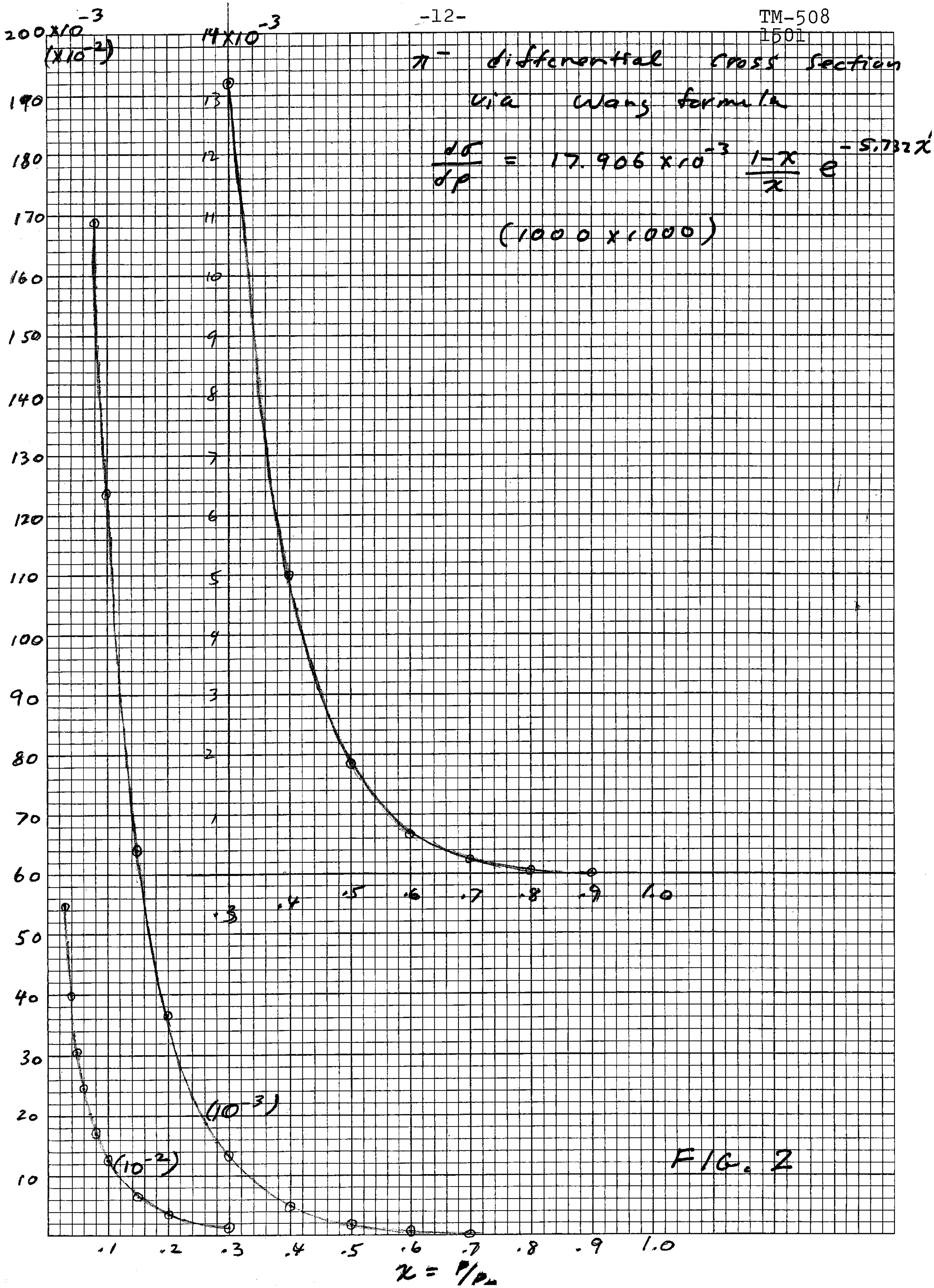

π^- differential cross section
via Wang formula
$\frac{d\sigma}{dp} = 17.906 \times 10^{-3} \frac{1-x}{x} e^{-5.732x}$
(1000 x 1000)
200×10^{-3}
$(\times 10^{-2})$
14×10^{-3}
(10^{-3})
(10^{-2})
$\frac{d\sigma}{dp}$ (mb/GeV/c)
$x = p/p_0$

FIG. 2

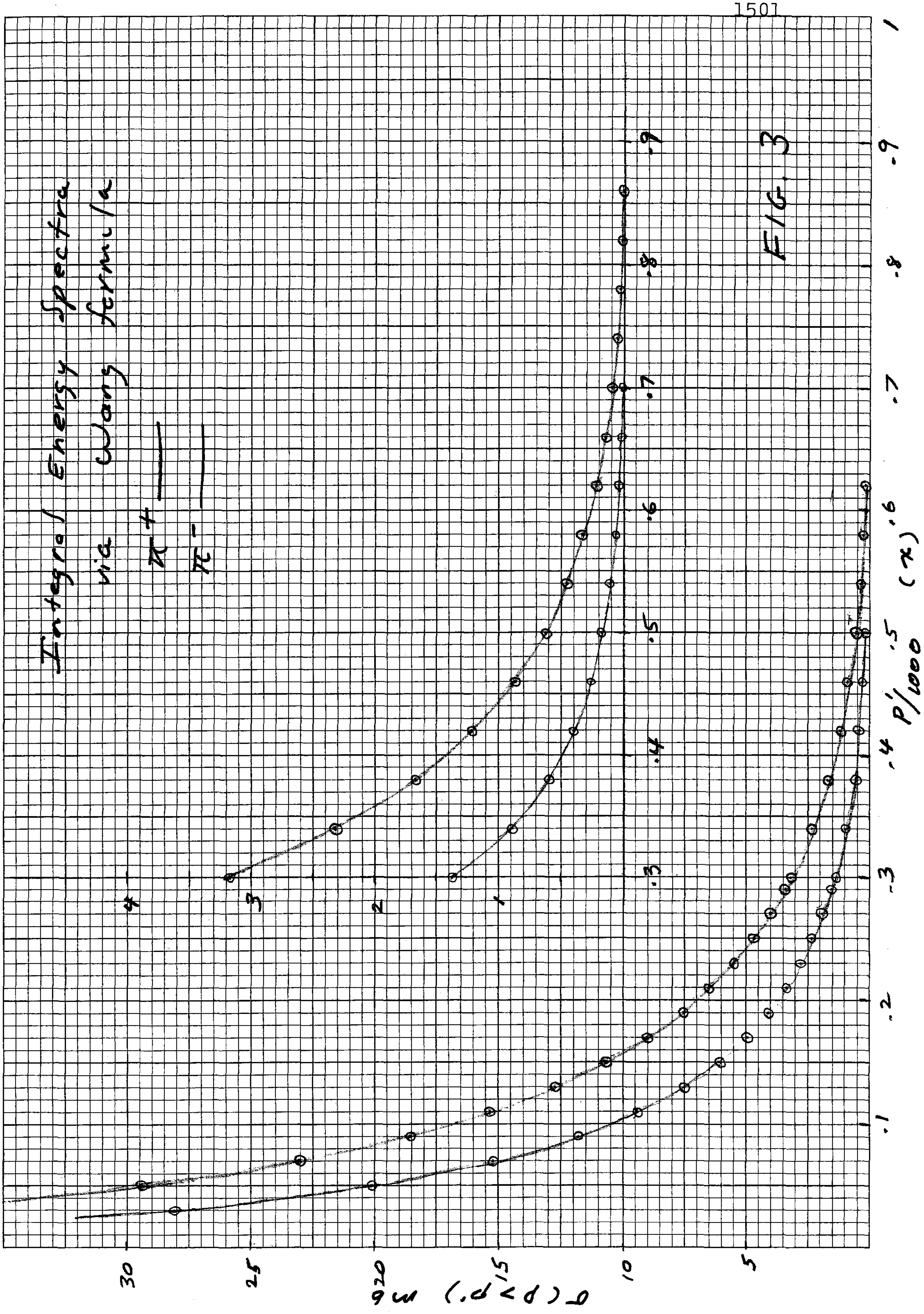
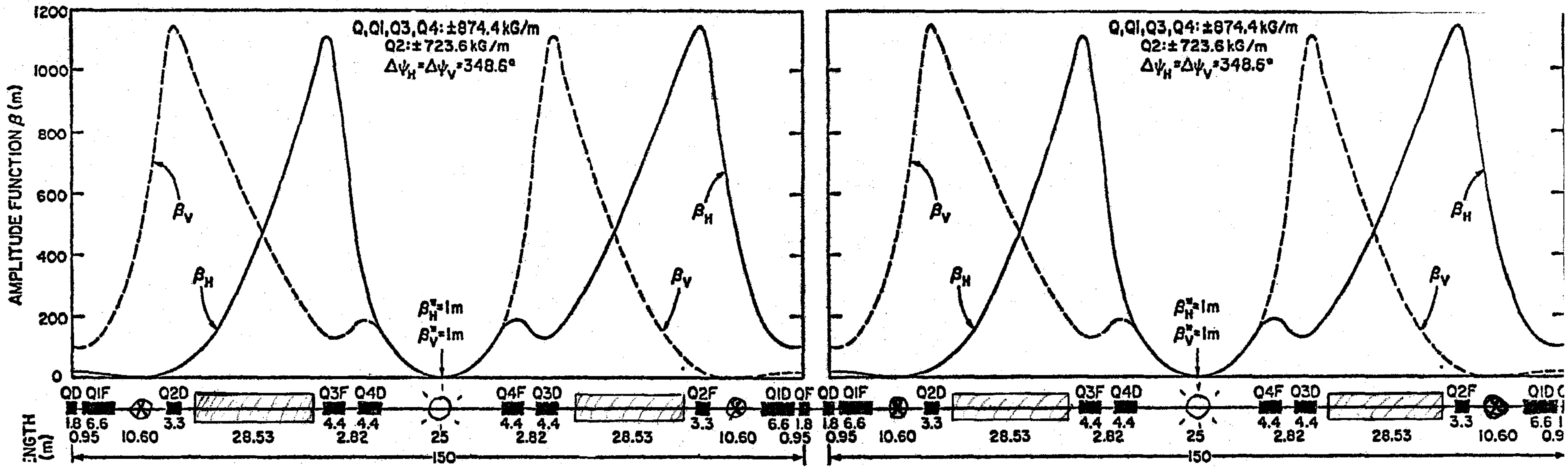

Integral Energy Spectra
via Wang formula
π+
π-
P'/1000 (π)
σ(p>p') mb
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
5
10
15
20
25
30

FIG. 3



⊗ Auxiliary Crossing

Experimental Crossing

Iron Shield (concentric with vaccum pipe)

Fig. 4

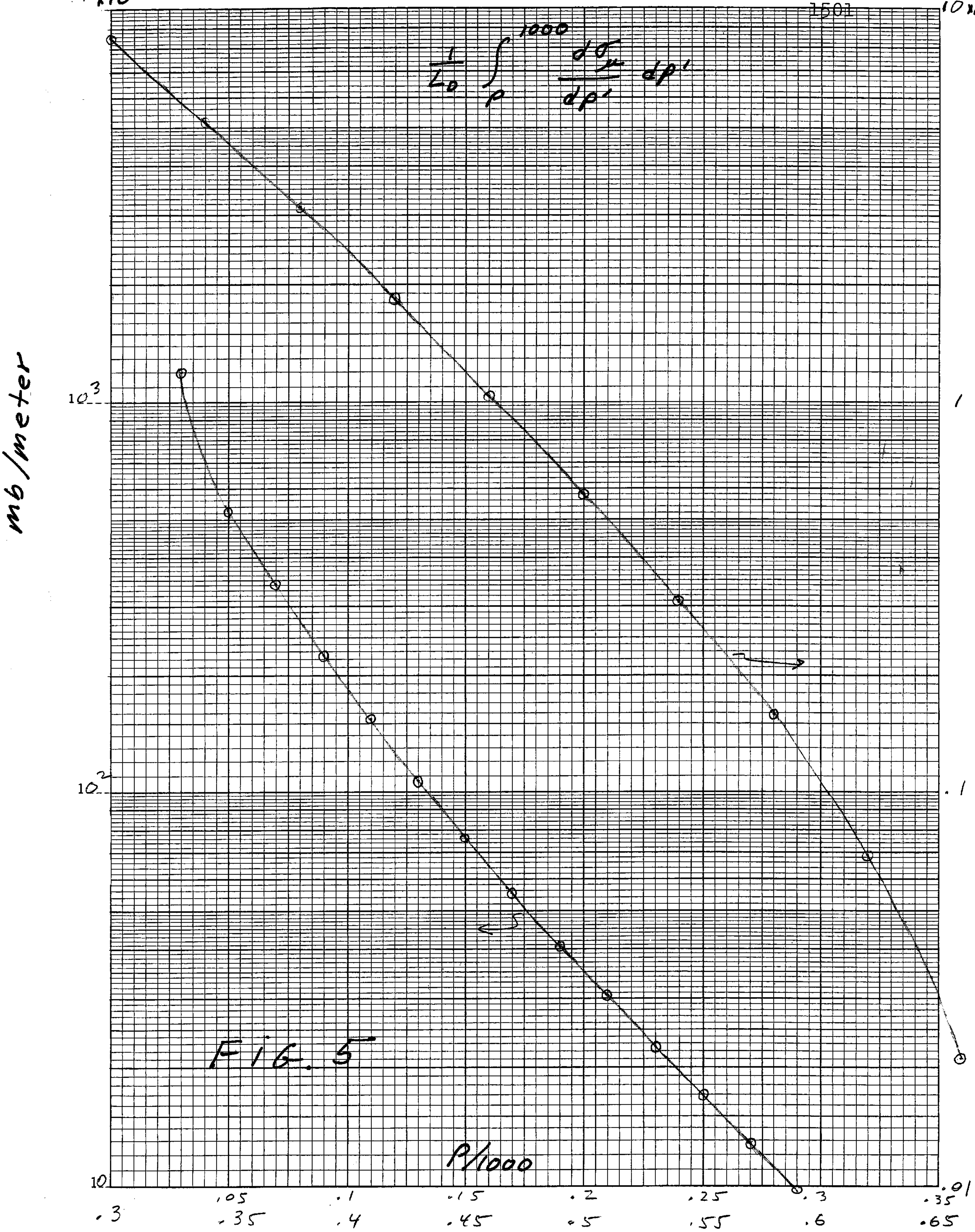

4×10^{-5}
10×1
$\frac{1}{L_0} \int_p^{1000} \frac{d\sigma_\mu}{dp'} dp'$
mb/meter
10^3
1
10^2
.1
10
.01
P/1000
.05
.1
.15
.2
.25
.3
.35
.3
.35
.4
.45
.5
.55
.6
.65

FIG. 5